EXPECTED QUESTIONS TO BE ASKED IN 2018

CLASS - XII **SUB : MATHEMATICS**

Very Short Answers Questions (1 Marks)

1. If f: R → R be given by f (x) = $(3-x^3)^{\frac{1}{3}}$, find the value of $f o f$ (x) Ans: x
2. Find fog ,if f(x)= $8x^3$ and g(x) = $x^{1/3}$. Ans: 8x
3. What is the principal value of co^{s-1}(cos 2π/3) +sin^{-1}(sin 2π/3) ? Ans: $^{\pi}$
4. Form a 2X3 matrix A = [a_{ij}], where $a_{ij}$is given by$\frac{|2i-j|}{3j}$ Ans:$\begin{bmatrix} 1/3 & 0 & 1/9 \\ 1 & 1/3 & 1/9 \end{bmatrix}$
5. For what value of x,is the following matrix singular ?
   $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ Ans:1
6. Let A be the square matrix of order 3x3,Find the value of |kA| Ans:k^3|A|
7. Let A be the non singular matrix of order 3x3, Find the value of |adjA| Ans: $|A|^2$
8. Let A is an invertible square matrix of order 2, find the Value of det (A^{-1}) Ans: 1/|A|
9. If A is a square matrix of 3 such that |adjA|=64 Find |A| Ans:8
10. If y = $\tan^{-1}$ (x) + $\tan^{-1}$ (1 / x), find $\frac{dy}{dx}$ Ans:0
11. Discuss the continuity of the function f(x) = 1/x. Ans: continuous everywhere
12. Find the derivative of $e^{\sqrt{\tan x}}$ Ans:$e^{\sqrt{\tan x}}$ $\sec^2$x/2√tan x
13. If x = cos t and y = sin t find $\frac{dy}{dx}$ Ans: - cot t
14. For y = e^{-mx}, find second order derivative of y w.r.t x. Ans: m^2e^{-mx}
15. Find slope of tangent to the curve y=x^2- 2 , when x=2. Ans:4
16. Find rate of change of area of a circle with respect to its radius when the radius is 2 Cm. Ans: 4π
17. Show that the function y=logx is strictly increasing in its domain.
18. Find the point on the curve y=x^2-2x at which tangent is parallel to X axis. Ans: (1,-1)
19. Find slope of normal to the curve y=x^3-3 , when x=$\frac{1}{\sqrt{3}}$. Ans: -1
20. Find the intervals in which y=x^3-12 is increasing or decreasing. Ans: Increasing in R
21. Find rate of change of volume of a cube with respect to its side, when the side is 3 cm. Ans:27
22. Evaluate ∫x^2/(1+x^3)dx Ans: 1/3 log (1+x^3)+c
23. Evaluate ∫$\tan^2$xdx Ans:tan x –x +c
24. Evaluate ∫(cosx-sinx)/(sinx+cosx)dx Ans: log (sinx+cosx)
25. Evaluate ∫dx/(x^2+25) Ans: 1/5 $\tan^{-1}$(x/5)
26. Evaluate$\int_0^1 \frac{tan^{-1}x}{1+x^2}dx$ Ans:1/2
27. Evaluate $\int_{-\pi/2}^{\pi/2} sin^7 x\, dx$ Ans:0
28. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}}$ + y = 0. Ans: order 2 and degree 4
29. If $\vec{a} = (5\hat{i} - \hat{j} - 3\hat{k})$ & $\vec{b} = (\hat{i} + 3\hat{j} - 5\hat{k})$, then show that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are ⊥ .

30. Find the projection of vector $\vec{a} = (2\hat{i} + 3\hat{j} + 2\hat{k})$ *on the vector* $\vec{b} = (\hat{i} + 2\hat{j} + \hat{k})$. Ans:10/√6

31. Find the intercepts cut off by the plane 2x + y - z =5. Ans: 5/2, 5,-5

<u>Short Answers Questions (4 Marks)</u>

1. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(*a*, *b*) :*b* = *a* + 1} is reflexive, symmetric or transitive. Ans: none.
2. Show that the function f:R→R define by f(X) = (2x-1)/3 , x ∈R is one-one and onto function. Also find the Inverse of the function f. Ans: $f^{-1}(y) = \frac{3y+1}{2}$
3. Let * be a binary operation on the set Q of rational numbers as follows :

   (i) a * b = $a^2 + b^2$ (ii) a * b = a + ab (iii) a * b = $\frac{ab}{4}$ (iv) a * b = a – b

   Find which of the binary operations are commutative and which are associative.

   Ans: (i) commutative, not associative (ii) neither commutative nor associative (ii) commutative and associative(ii) not commutative, not associative

4. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \beta$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\beta + \frac{y^2}{b^2} = \sin^2\beta$
5. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$
6. Prove that $\tan^{-1}(63/16) = \sin^{-1}(5/13) + \cos^{-1}(3/5)$
7. Prove the following

   $\cot^{-1}\left(\frac{\sqrt{(1+\sin x)}+\sqrt{(1-\sin x)}}{\sqrt{(1+\sin x)}-\sqrt{(1-\sin x)}}\right) = \frac{x}{2}$
8. Solve for x

   $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ Ans: $\pi/4$
9. Prove that

   $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

10. Using properties of determinants, Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

11. Using properties of determinants prove that

    $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = abc + ab +bc +ca

12. Using properties of determinants prove that

    $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

13. If A = $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$, find A^{-1} using elementary row transformations. Ans: 1/8$\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

14. Find the value of k so that the function f(x)= $\begin{cases} \frac{1-\cos 4x}{8x^2} & if x \neq 0 \\ 3k & if x = 0 \end{cases}$ is continuous at x= 0. .

Ans:1/3

15. If y = a sin 2t (1 + cos 2t) and y = b cos 2t (1- cos2t), then find the value of $\frac{dy}{dx}$.

16. If y=Ae ^{mx}+Be^{nx} ,prove that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$.

17. Find $\frac{dy}{dx}$ if $x^y = y^x$. Ans: $\frac{dy}{dx}=\frac{y(xlogy-y)}{x(ylogx-x)}$

18. If y = $e^{\tan^{-1}x}$ Prove that $(1+x^2)y_2+(2x-1)y_1=0$. y2and y1 are second and first derivatives of y wrt x.

19. If y =$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ find $\frac{dy}{dx}$ Ans:1/2

20. If x = $\sqrt{a^{\sin^{-1}t}}$ and y =$\sqrt{a^{\cos^{-1}t}}$then prove that $\frac{dy}{dx}=-\frac{y}{x}$

21. If cos y = x cos(a+y) then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$

22. If y =$\sin^{-1}\frac{5x+12\sqrt{1-x^2}}{13}$then find $\frac{dy}{dx}$. Ans:$\frac{1}{\sqrt{(1-x^2)}}$

23. If y =$\tan^{-1}x$ show that $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$

24. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

25. If y = $\left[x+\sqrt{x^2+1}\right]^n$then show that $(x^2+1)y_2+xy_1-n^2y=0$

26. Show that the curves x= y ^{2} and x y=k cut at right angles , if $8k^2 = 1$.

27. Find the intervals in which y= - $2x^3-9x^2-12x+1$ is strictly increasing or strictly decreasing.
Ans: st.inc for -2<x<-1, stdec for x<-2 & x>-1

28. Find the interval in which f(x)= sin 2x where xϵπ is increasing or decreasing.

32. Verify Rolle's Theorem for the function f(x) = x^2-x-6 in the interval [-2,3] .

33. Find the Equation of tangent to the curve y =x^2-2x +7which is parallel to line 2x-y +9=0
Ans: 2x-y +4=0.

34. $\int_0^{\pi/2}\sin 2x\log(\tan x)\,dx$ Ans. 0

35. $\int(x^2+1)(x^2+2)/(x^2+3)(x^2+4)dx$
Ans-x+(2/√3)$\tan^{-1}$ (x/√3)-3 $\tan^{-1}$(x/2)+c

36. $\int\frac{(3\sin x-2)\cos x}{5-\cos 2x-4\sin x}dx$ Ans-3log(2-sinx)+4/(2-sinx)+c

37. Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$. Ans: $\frac{\pi^2}{4}$

38. Evaluate: $\int\frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx$ Ans: $\frac{1}{a^2-b^2}\log\left|a^2\sin^2 x+b^2\cos^2 x\right|+c$

39. Evaluate: $\int\frac{x^2}{x^2+6x+12}dx$. Ans: $x-3\log\left|x^2+6x+12\right|+\frac{6}{\sqrt{3}}\tan^{-1}\left[\frac{x+3}{\sqrt{3}}\right]+c$

40. Prove that : $\int_0^{\pi}\frac{x}{1+\sin x}dx=\pi$.

41. Evaluate $\int \frac{1}{3+2\sin x+\cos x}dx$. Ans: $\tan^{-1}(\tan\frac{x}{2}+1)+c$

42. Evaluate $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$ Ans: $-\frac{2}{b^2}\left[\log\left|a+b\cos x\right|+\frac{a}{a+b\cos x}\right]+c$

43. Evaluate $\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx$. Ans: $\frac{x}{\log x}+c$

44. Evaluate : $\int \frac{4x+5}{2x^2+8x+3}dx$. Ans: $\log|2x^2+8x+3|-\frac{3\sqrt{2}}{4\sqrt{5}}\log\left|\frac{\sqrt{2}(x+2)-\sqrt{5}}{\sqrt{2}(x+2)+\sqrt{5}}\right|+c$

45. Evaluate: $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$. Ans:1

46. Evaluate: $\int_0^1 \cot^{-1}(1-x+x^2)dx$. Ans: $\frac{\pi}{2}-\log 2$

47. Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$. Ans: $\pi(\frac{\pi}{2}-1)$

48. Evaluate: $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}dx$. Ans: $a(\frac{\pi}{2}-1)$

49. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}dx$. Ans: $\frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$

50. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$, using properties of definite integrals. Ans: $\frac{\pi}{8}\log 2$

51. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$. Ans: $\frac{\pi}{2}$

52. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$. Ans: $\frac{\pi}{12}$

53. Evaluate: $\int_{-5}^{0} f(x)dx$, where f(x)= $|x|+|x+2|+|x+5|$. Ans: $\frac{63}{2}$

54. Evaluate: $\int\sqrt{\tan\phi}\,d\phi$. Ans: $\frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan\phi-1}{\sqrt{2}\tan\phi})+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan\phi+1-\sqrt{2\tan\phi}}{\tan\phi+1+\sqrt{2\tan\phi}}\right|$+c

55. Solve $(x+y)\frac{dy}{dx}=1$ Ans: $x+y+1=ce^y$

56. Solve the following differential equation $(x^2+xy)dy=(x^2+y^2)dx$

Ans: $\log|x|-2\log|x-y|-\frac{y}{x}=c$

57. Solve $\frac{dy}{dx}=\cos(x+y)$ Ans: $\tan\left(\frac{x+y}{2}\right)=x+c$

58. Solve the differential equation: $\frac{dy}{dx}+y\cot x=2x+x^2\cot x$ Ans: $\left[y.\sin x=x^2\sin x+c\right]$

59. Find the angle between the planes whose vector equations are $\vec{r}$. (2 i+2j-3k) = 5 & $\vec{r}$. (3i-3j+5k)=3.
Ans: $\cos^{-1}\frac{15}{\sqrt{731}}$

60. Find the equation of the plane passing through the point (1, -1, 2) and perpendicular to the planes 2x + 3y - 2z = 5 and x +2y -3z = 8. Ans: 5x-4y-z=7

61. Find the co-ordinates of the point where the line through (3, -4, 5) and (2, -3, 1) crosses the plane 2x + y + z = 7. Ans :(1,-2.7)

62. Show that the points A (1, 2, 7), B (2, 6, 3) & C (3, 10, -1) are collinear.

63. Find the angle between the lines: $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ *and* $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$. Ans: $\cos^{-1}\frac{8\sqrt{3}}{15}$

64. Find S.D. between the lines:

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k} \text{ and}$$
$$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$$

Ans: $\frac{8}{\sqrt{29}}$

65. Find the Cartesian equation of the line passing through the point (-2, 4, -5) and parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Ans: $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$.

66. Find the value of p so that the lines, $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.
Ans: p=$\frac{70}{11}$

67. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3,\ |\vec{b}|=4,\ |\vec{c}|=5$ and each one of them being perpendicular to the sum of other two, Find $|\vec{a}+\vec{b}+\vec{c}|$. Ans: $5\sqrt{2}$

68. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

69. If $\vec{a}=(\hat{i}+\hat{j}+\hat{k}), \vec{b}=(2\hat{i}-j+3\hat{k})$ & $\vec{c}=(\hat{i}-2j+\hat{k})$ find a unit vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$.
Ans: $\frac{3}{\sqrt{22}}\hat{\imath}-\frac{3}{\sqrt{22}}\hat{\jmath}+\frac{2}{\sqrt{22}}\hat{k}$

70. Let $\vec{a}=(\hat{i}+4\hat{j}+2\hat{k}), \vec{b}=(2\hat{i}-2j+7\hat{k})$ & $\vec{c}=(2\hat{i}-j+4\hat{k})$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ & $\vec{b}$ *and* $\vec{c}.\vec{d}=15$. Ans : $\frac{1}{3}(160\hat{i}-5\hat{j}+4\hat{k})$

71. Show that the points A, B and C with position vectors, $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively, form the vertices of a right angled triangle.

72. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ = 0, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$. Ans: -$\frac{-3}{2}$

73. The two adjacent sides of a parallelogram are $(2\hat{i}-4\hat{j}+5\hat{k})$ & $(\hat{i}-2j-3\hat{k})$. Find the unit vectors parallel to its diagonals. Also find its area. Ans: $\frac{1}{3}(3\hat{i}-6\hat{j}+2\hat{k}), 11\sqrt{5}$

74. If $(\hat{i}+\hat{j}+\hat{k}),(2\hat{i}+5\hat{j}-3\hat{k}),(3\hat{i}+2\hat{j}-2\hat{k})\,\&\,(\hat{i}-6j-\hat{k})$ are the position vectors of points A, B, C & D respectively, then find the angle between AB & CD. Deduce that AB& CD are parallel. Ans: $\cos^{-1}\frac{5}{\sqrt{187}}$

75. Find the value of λ such that the vectors $(3\hat{i}+\lambda\hat{j}+5\hat{k}),(\hat{i}+2\hat{j}-3\hat{k})\,\&\,(2\hat{i}-j+\hat{k})$ are coplanar.

76. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

77. The probability of a man hitting a target is ½. How many times must he fire so that the probability of hitting the target atleast once is more than 90%. Ans: $n\geq 4$

78. A coin is tossed 5 times .what is the probability of getting at least 3 heads. Ans:1/2

79. A random variable X has the following probability distribution find (i) k (ii) P(X $\leq$ 1) (iii) P(X>3)

X	0	1	2	3	4	5
P(X)	0.1	K	0.2	2k	0.3	K

80. An unbiased coin is tossed six times. Find the probability of getting
    a. at the most two heads b. more than 4 heads

81. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine (i) k (ii) p(X<3) (iii) p(X>6) (iv) p(0<X<3) Ans:1/10,3/10,17/100,3/10

82. . The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none (ii) not more than one (iii) at least one will fuse after 150 boys?
Ans: $0.95^5, 0.95^4*1.2, (1-0.95^5)$

83. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of (i) 5 successes? (ii) at least 5 successes? (iii) at most 5 successes? Ans:3/32,7/64,63/64

84. There are three urns A, B and C. Urn A contains 4 white and 5 blue balls. Urn B contains 4 white and 3 blue balls. Urn C contains 2 white and 4 blue balls. One ball is drawn from each bag. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?

85. A speaks truth in 55% cases and B speaks truth in 75% cases. Find the % of cases in which they are likely to contradict each other in stating the same fact? Ans:19/40

86. Bag A contains 3 white and 2 red balls. Bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags.

    (i) find the probability that the ball drawn from the bag is red,

    (ii) if the ball drawn is found to be red, find the probability that it was drawn from the bag B.

87. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die?

88. There are two bags. The first bag contains 4 white and 2 black balls, while the second bag contains 3 white and 4 black balls. A bag is picked up at random and a ball is drawn out. Find the probability that it is a white ball.

89. In a group of 9 students, there are 5 boys and 4 girls. A team of 4 students is to be selected for a quiz competition. Find the probability that there will be 2 boys and 2 girls in that team.

90. Each of A and B throw two dice simultaneously turn by turn. A will win if he throws a total of 5, B will win if he throws a doublet. Find the probability that B will win the game though A started it.

Long Answers Questions (6 Marks)

1. Show that the relation R in the set A= { x: x ∈ W, $0 \le x \le 12$ } given by R={ (a,b); (a-b) is multiple of 4 }is an equivalence relation. Also find the set of all elements related to 2
2. Consider f : $R^+ \to [-5, \infty)$ given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible and

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$$, where R^+ is the set of all non-neg. real numbers.

3. Prove that $\frac{1}{2}\tan^{-1} x = \cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right)$
4. Using Matrix solve the following system of equations
   x+y+z = 6 , 2x+3y+3z =17 , x-y+z=2 Ans: x=1, y=2 and z=3
5. Express $\begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 5 \\ 4 & 6 & 1 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix.
6. solve the system of equations using Matrices
   $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$
7. Using elementary operation , find inverse of matrix= $\begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 3 \\ 4 & 1 & 5 \end{bmatrix}$
8. If a, b, c are all positive and are p^{th} , q^{th} and r^{th} terms of G.P., then find the value of
   $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$
9. Let A = $\begin{bmatrix} 0 & -\tan\frac{r}{2} \\ \tan\frac{r}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2x2, show that :
   I + A = (I-A) $\begin{bmatrix} \cos r & -\sin r \\ \sin r & \cos r \end{bmatrix}$
10. If y = a (cos t + log tan t/2) and x = a sin t then find $\frac{d^2y}{dx^2}$ when t = π / 4.
11. For $y = e^x \tan^{-1} x$ then show that $(1 + x^2)\frac{d^2y}{dx^2} - 2(1 - x + x^2)\frac{dy}{dx} + (1-x)^2 y = 0$
12. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$
13. If $2x = y^{1/5} + y^{-1/5}$ then express "y" as an explicit function of x and prove that
    $$(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 25y$$
14. If $y = \sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$, find $\frac{dy}{dx}$ Ans:0
15. Find the value of k so that the function f(x)= $\begin{cases} \frac{k\cos x}{\pi - 2x} & if x \neq \frac{\pi}{2} \\ 3 & if x = \frac{\pi}{2} \end{cases}$ is continuous at x= $\frac{\pi}{2}$. Ans:6

16. Show that the area of a right-angled triangle of a given hypotenuse is maximum when the triangle is isosceles.
17. Show that the isosceles triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
18. Sum of the perimeter of a square and a circle is constant. Show that the sum of their area is least when the side of the square is equal to the diameter of the circle.
19. A cylinder is such that sum of its height and the circumference of the base is 10m. Find the greatest volume of the cylinder.
20. Show that the right circular cone of least curved surface and given volume has altitude equal to $\sqrt{2}$ times the radius of the base.
21. A given quantity of metal to be cast into half cylinder with a rectangular base and semicircular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of its semi-circular ends is $\pi : \pi + 2$.
22. Find the cylinder of the greatest volume that can be inscribed in a given right circular cone.
23. Show that semi vertical angle of a cone of maximum volume and given slant height is $\mathbf{\tan^{-1}\sqrt{2}}$.
24. A window is in the form of a rectangle surmounted by a semicircular opening . The total perimeter of the window is 20 m . find the dimensions of the window so as to allow the maximum light into the room.
25. Evaluate $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$, $\beta > \alpha$, Ans. $2\sin^{-1}\sqrt{\frac{(x-\alpha)}{(\beta-\alpha)}} + C$
26. Evaluate $\int \frac{\sec^2 x\, dx}{(\sec x+\tan x)^{9/2}}$,

    Ans. $-[\frac{1}{7(\sec x+\tan x)^{7/2}} + \frac{1}{11(\sec x+\tan x)^{1/2}}] + C$
27. Evaluate: $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx$. Ans $\frac{\pi}{2} - \log 2$
28. $\int_{-\pi/2}^{\pi/2}(\sin|x| + \cos|x|)dx$ Ans. 4
29. $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$ Ans.$2\sin^{-1}\frac{\sqrt{3}-1}{2}$
30. Evaluate the integral: $\int \frac{\sin x}{\sin 4x}dx$
31. Find $\int_0^1 x(\tan^{-1} x)^2\, dx$ Ans: $\frac{\pi^2}{16} - \frac{\pi}{4} + \log\sqrt{2}$
32. Find the area of region given by $\{(\mathbf{x}, \mathbf{y}: x^2 \le y \le |x|\}$

    Ans- 1/3 Sq. Unit.
33. Find the area of region given by $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$ Ans- (π/2 -1/2) Sq. Unit.
34. Find the area lying above X axis and included between the circle $x^2+y^2 =8x$ and the parabola $y^2 = 4x$. Ans.4/3(8+3π) sq units.
35. Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2+y^2 =1$.

    Ans.2π/3-√3/2 sq units .
36. Find the area of the region bounded by the curves $y^2=4x$ and $4x^2+4y^2=9$.

    Ans.9 π/8-9/4 sin-11/3+1/3√2
37. Find the area bounded by the curves $x^2=y$, y=x and y=-x .

    Ans.1/3 sq units.
38. Using integration find the area of the region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1). Ans.3/2 sq units.
39. Find the area of region given by $\{(x, y): y^2 \ge 6x \text{and } x^2 + y^2 \le 16\}$
40. Find the area of region $\{(x, y): y \ge x^2 \text{and } y = |x|\}$
41. Solve: $ye^y dx = (y^3 + 2xe^y)dy$ Ans: $\left[x = -y^2e^{-y} + cy^2\right]$
42. Solve:$(x^2 - y^2)dx+2xy\, dy=0$, at x=1 and y=1. Ans:$x^2+y^2=2x$
43. Solve the differential equation $\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$, at x=1 and y=1

Ans: $\log|x|=-\frac{1}{2}\log\left|\frac{2y^2-xy+x^2}{x^2}\right|-\frac{3}{\sqrt{7}}\tan^{-1}\frac{4y-x}{\sqrt{7}x}+\frac{1}{2}\log 2+\frac{3}{\sqrt{7}}\tan^{-1}\frac{3}{\sqrt{7}}$.

44. Solve the differential equation: $\cos^2 x\frac{dy}{dx}+y=\tan x$

Ans: $y=\tan x-1+ce^{-\tan x}$

45. Solve: $(1+e^{2x})dy+(1+y^2)e^x dx=0$, given that x=0,y=1.

Ans: $e^x y=1$

46. Solve: $x\frac{dy}{dx}$ -ydx= $\sqrt{x^2+y^2}$ dx Ans: $y+\sqrt{x^2+y^2}=cx^2$

47. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0, \vec{r}.(2\hat{i}+j-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

48. Find the distance of the point (6, 5, 9) from plane passing through the points (3, -1, 2), (5, 2, 4) and (-1, -1, 6).

49. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

50. Find the equation of the plane passing through the line of intersection of the planes 2x+y-z = 3, 5x-3y+4z+9=0 and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$

51. Find the equation of the plane passing through the point (-1,3, 2) and perpendicular to the planes x +2y +3z = 5 and 3x +3y + z = 0.

52. Find the co-ordinates of the point where the line through (3, 4, 1) and (5, 1, 6) crosses the xy-plane .

53. Find the S.D. between: $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also find the equation of shortest distance.

54. A grain dealer has Rs.1500 for purchase of rice and wheat .A bag of rice and a bag of wheat costs Rs.180 and Rs.120 respectively. He has a storage capacity of 10 bags only. He earns a profit of Rs.11 and Rs.8 per bag of rice and wheat respectively. How many bags of each must he buy to make maximum profit?

55. A diet is to be contain at least 80 units of vitamin A and 100 units of minerals. Two foods P and Q are available. Food P costs Rs.4 per unit and food Q costs Rs.6 per unit. One unit of food P contains 3 units of vitamin A and 4 units of minerals. One unit of food Q contains 6 units of vitamin A and 3 units of minerals. Formulate this as L.P.P. and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

56. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs.5760 to invest and has a space for at most 20 items. A fan costs him Rs.360 and a sewing machine Rs.240. His expectation is that he can sell a fan at a profit of Rs.22 and a sewing machine at a profit of Rs.18.Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize the profit? Formulate this as a linear programming problem and solve it graphically.

57. A medical company has factories at two places A and B. From these places, supply is made to each of its three agencies situated at P, A and R. The monthly requirements of the agencies are respectively 40, 40 and 50 packets of the medicines, while the production capacity of the factories A and B are 60 and 70 packets respectively. The transportation costs per packet from the factories to the agencies are given below:

Transportation cost per packet		
(in rupees)		
To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory should be transported to each agency so that the cost of transportation is minimum? Also find the minimum cost.

58. One kind of cake requires 200g of flour and 25 gram of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.
59. A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 5% phosphoric acid and type B which contains 6% nitrogen and 10% of phosphoric acid . After testing the soil conditions of the field , it was found that at least 14kg of nitrogen and 14kg of phosphoric acid is required for producing a good crop. The fertilizer of type A costs Rs.5 per kg and the type B costs Rs.3 per kg .How many kg of each type of fertilizer should be used to meet the requirement at the minimum possible cost? Using L.P.P solve above problem graphically.
60. Find the mean, variance and standard deviation of the number of heads in a simultaneous tosses of three coins.
61. A class has 14 students whose ages are 4,7,5,4,6,7,9,6,8,6,5,4,4,9 years. One student is selected at random and the age x of him is recorded. What is the probability distribution of random variable x? Find mean, variance and standard deviation of x.
62. A factory, has three types of machines X, Y and Z producing 2000, 4000 and 6000 bolts per day respectively. The machine X produces 1% defective bolts, machine Y produces 1.5% and machine Z produces 2% defective bolts. At the end if a day, a bolt is picked at random and is found to be defective. Find the probability that this defective blot is not produced by the machine X. Ans:9/10
63. A man is known to speak truth 5 out of 7 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Ans:1/3
64. There are three coins .One is a two headed coin (having head on bothfaces) another is a biased coin that comes up tails 25% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed it shows heads,what is the probability that it was the two headed coin ? Ans:4/9
65. A factory, has three types of machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, machine Y produces 1.5% and machine Z produces 2% defective bolts. At the end if a day, a bolt is picked at random and is found to be defective. Find the probability that this defective blot is produced by the machine X. Ans:1/10
66. An insurance company insured 2000 scooter drivers, 3000 car drivers and 5000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.03, 0.04 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he was a truck driver? Ans:25/31